

Suffolk Public Schools
AP Calculus AB
Summer Assignment 2019

Summer Assignment:

Suffolk Public Schools wishes to promote a summer assignment that prepares students for a rigorous program in Calculus. The following assignments are designed to promote a serious academic attitude that demonstrates a high degree of self-discipline and self-motivation.

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Resources:

As you complete your summer assignment, you may need to review math concepts and rules you learned in previous courses. Try these resources:

The website <http://www.math.vanderbilt.edu/~pscrooke/calculus/AT1.pdf> will direct you to a very thorough review of Algebra and Trigonometry material in order to prepare you for Calculus. It has a summary of important topics as well as practice problems and answers to odd-numbered problems. The following website <http://www.math.vanderbilt.edu/~pscrooke/calculus/calculus.html> has a number of other helpful resources.

Two additional general websites for math tutorials and practice are Kahn Academy, <http://www.khanacademy.org/>.

If you cannot find adequate assistance on your own, through these online resources, contact your teacher by email – the email address for the AP Calculus teacher at each high school are listed below.

Suffolk Public Schools Instructors

Lakeland High School: Rachele Mogren, rachellemogren@spsk12.net (Join Remind--Class Code: hg42ce

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King's Fork High School: TBD

The AP Calculus summer assignment consists of 2 parts—Short Answer and Multiple Choice. Please complete each section in its entirety. **Part 1 and Part 2 are due the second day the class meets in September.**

Summer Assignment –Part 1

Directions: Please read questions carefully. Show all work on this packet. If no work is required, explain how you arrived at your answer. Follow the calculator instructions as given in each section.

These problems are due the **2nd day the class meets** in September. Please do your best to fully understand all the material in this packet.

Short Answer Section

Objectives: For objectives #1 - #5 you should be able to do all without a calculator

1. Identify functions as even or odd

Algebraically

Graphically

2. Know key points and basic shapes of essential graphs

$$f(x) = \sqrt{x}, f(x) = x^2, f(x) = x^3$$

$$f(x) = e^x$$

$$f(x) = \ln(x)$$

$$f(x) = \sin x, f(x) = \cos x$$

3. Review Trigonometry

Evaluate trig values

Evaluate inverse trig values

Solve trig equations

4. Understand characteristics of rational expressions (be able to sketch **without** a calculator)

Vertical asymptotes

Horizontal asymptotes

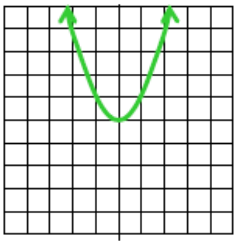
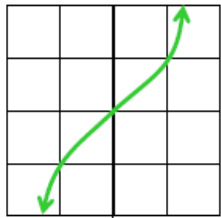
Zeros

Holes

5. Be able to use properties of natural logarithms to solve equations

6. Know your calculator

I. Symmetry – Even and Odd Functions

Quick Review		
Even Function	Symmetric about the y axis $f(-x) = f(x)$ for all x	Example: $y = x^2$ 
Odd Function	Symmetric about the origin (equivalent to a rotation of 180 degrees) $f(-x) = -f(x)$ for all x	Example: $y = x^3$ 

To determine algebraically if a function is even, odd, or neither, find $f(-x)$ and determine if it is equal to $f(x)$, $-f(x)$, or neither.

Example: Determine if $f(x) = \frac{4x}{x^2+1}$ is even or odd.

$$f(-x) = \frac{4(-x)}{(-x)^2+1} = \frac{-4x}{x^2+1} = -\frac{4x}{x^2+1} = -f(x) \text{ Therefore, } f(x) \text{ is an odd function.}$$

Determine if the following functions are even, odd, or neither.

1. $f(x) = \frac{x^2}{x^4+3}$

2. $f(x) = \frac{x}{x+1}$

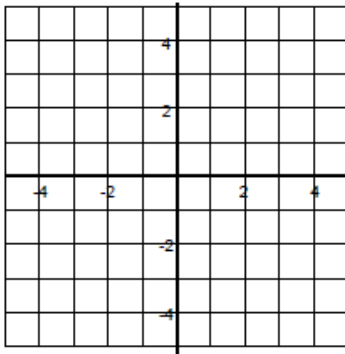
3. $f(x) = 1 + 3x^2 + 3x^4$

4. $f(x) = 1 + 3x^3 + 3x^5$

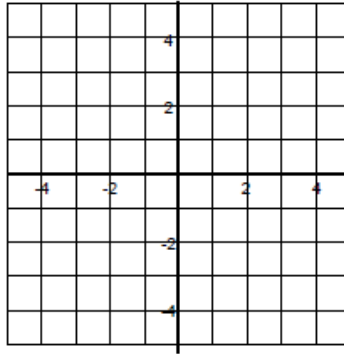
II. Essential Graphs

For each graph, show two key points (label coordinates) and basic shape of the graph.

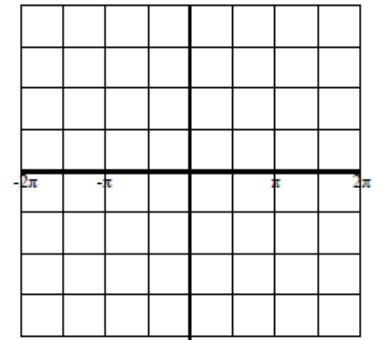
1. $f(x) = \sqrt{x}$



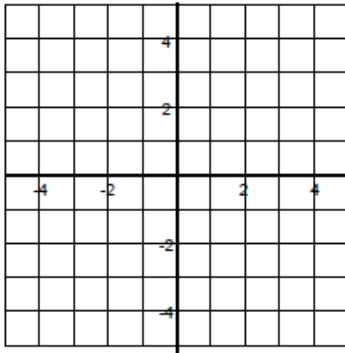
2. $f(x) = x^3$



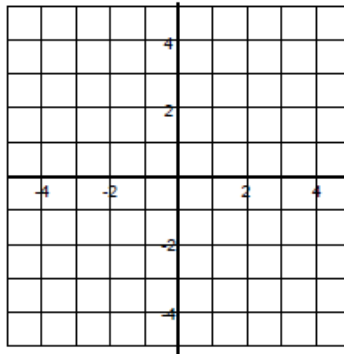
3. $f(x) = \sin x$



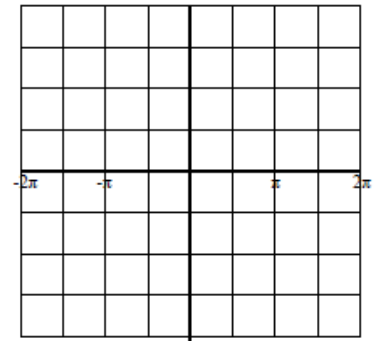
4. $f(x) = x^2$



5. $f(x) = \frac{1}{x}$

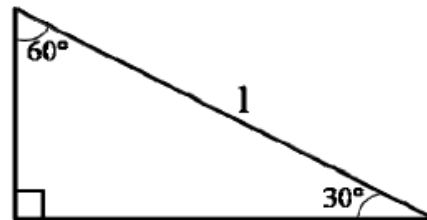
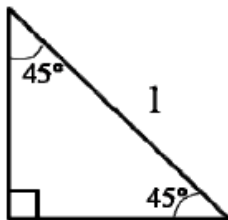


6. $f(x) = \cos x$



III. Trigonometry Review

On each right triangle with a hypotenuse of 1, label the lengths of the other sides:



$45^\circ = \underline{\hspace{2cm}}$ radians

$90^\circ = \underline{\hspace{2cm}}$ radians

$30^\circ = \underline{\hspace{2cm}}$ radians

$60^\circ = \underline{\hspace{2cm}}$ radians

⇒ In calculus we always use radians! Never degrees.

Make sure you know the following identities:

Pythagorean identities: $\sin^2 x + \cos^2 x = 1$ $1 + \tan^2 x = \sec^2 x$ $\cot^2 x + 1 = \csc^2 x$

Double angle identities: $\sin 2x = 2 \sin x \cos x$ $\cos 2x = \cos^2 x - \sin^2 x$

1. Evaluate the sine, cosine, and tangent of each angle without using a calculator. It is not necessary (ever again!) to rationalize the denominator. (ie, an answer of $\frac{1}{\sqrt{2}}$ does not need to be written $\frac{\sqrt{2}}{2}$)

a) $\frac{\pi}{4}$

b) $\frac{5\pi}{4}$

c) $-\frac{\pi}{6}$

d) $\frac{\pi}{2}$

e) $\frac{5\pi}{3}$

f) $\frac{11\pi}{6}$

2. Find two solutions to the inverse trig equations. ($0 \leq \theta \leq 2\pi$).

1. Draw a triangle, if necessary
2. Label θ and sides
3. Determine reference angle
4. Notice restrictions
5. Place in correct quadrant

a) $\cos \theta = \frac{\sqrt{2}}{2}$

b) $\cos \theta = -\frac{\sqrt{2}}{2}$

c) $\sec \theta = 2$

d) $\tan \theta = 1$

e) $\cot \theta = -\sqrt{3}$

f) $\sin \theta = -\frac{\sqrt{3}}{2}$

3. Solve for y . Notice the only difference between question #2 and #3 is the domain restrictions #3. Inverse trig functions have one answer due to domain restrictions. Note: $\sin^{-1}x$ is the same as $\arcsin x$

a) $y = \sin^{-1}\left(\frac{1}{2}\right)$

b) $y = \cos^{-1}\left(\frac{1}{2}\right)$

c) $y = \arctan(\sqrt{3})$

d) $y = \csc^{-1}(2)$

e) $y = \arccos(0)$

f) $y = \cot^{-1}(-1)$

IV. Rational Functions

Rational functions are ratios of polynomials: $h(x) = \frac{f(x)}{g(x)}$

$h(x)$ has a **zero** when $h(x) = 0$ (which occurs when $f(x) = 0$ and the factor does not cancel)

$$\text{Ex. } h(x) = \frac{x^2 + x - 2}{x^2 - 1} \quad h(x) = \frac{(x+2)(x-1)}{(x+1)(x-1)} \quad h(x) = \frac{\cancel{(x+2)}\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$$

Therefore, $h(x) = 0$ when $x = -2$.

$h(x)$ has a **vertical asymptote** when $g(x) = 0$ and the factor that causes $g(x) = 0$ does not cancel

$$\text{Ex. } h(x) = \frac{x^2 + x - 2}{x^2 - 1} \quad h(x) = \frac{(x+2)(x-1)}{(x+1)(x-1)} \quad h(x) = \frac{\cancel{(x+2)}\cancel{(x-1)}}{\cancel{(x+1)}(x-1)}$$

Therefore, $h(x)$ has a vertical asymptote when $x = -1$.

$h(x)$ has a **hole** (is undefined but the limit exists) when $g(x) = 0$ and the factor that causes $g(x) = 0$ cancels from both $f(x)$ and $g(x)$.

$$\text{Ex. } h(x) = \frac{x^2 + x - 2}{x^2 - 1} \quad h(x) = \frac{(x+2)(x-1)}{(x+1)(x-1)} \quad h(x) = \frac{(x+2)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$$

Therefore, $h(x)$ has a hole when $x = 1$.

Note that $h(x) \neq \frac{(x+2)}{(x+1)}$ because these two functions do not have the same domain.

$h(x)$ has a **horizontal asymptote** at $y = a$ when $\lim_{x \rightarrow \infty} h(x) = a$ or $\lim_{x \rightarrow -\infty} h(x) = a$. To determine $\lim_{x \rightarrow \infty} h(x)$ consider first the largest exponent of $f(x)$ and $g(x)$. If $f(x)$ has the larger exponent, then $\lim_{x \rightarrow \infty} h(x) = \infty$ (DNE). If $g(x)$ has the larger exponent, then $\lim_{x \rightarrow \infty} h(x) = 0$. If the exponents are the same, consider the leading coefficient.

$$\text{Ex. } h(x) = \frac{x^2 + x - 2}{x^2 - 1} \quad \text{Leading coefficients} = \frac{1}{1}$$

Therefore, $\lim_{x \rightarrow \infty} h(x) = 1$ and $h(x)$ has a horizontal asymptote at $y = 1$.

Once the basic characteristics of rational expressions are determined, the functions can be sketched without a calculator:

Ex. $h(x) = \frac{x^2 + x - 2}{x^2 - 1}$

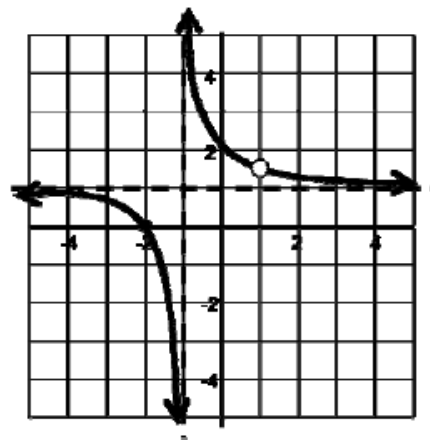
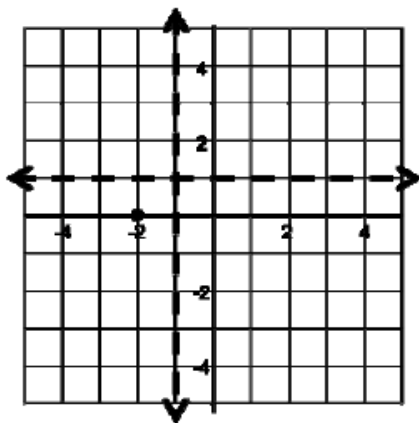
Zero at $x = -2$.

Vertical Asymptote: $x = -1$

Hole when $x = 1$

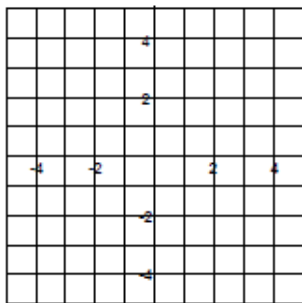
Horizontal Asymptote: $y = 1$

Graph points as needed until you see the shape.

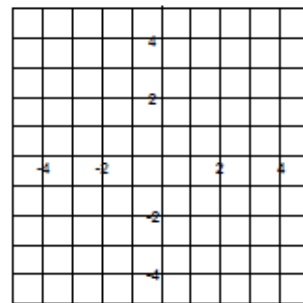


- For each rational function below:
- Find all zeros
 - Write the equations of all vertical asymptotes
 - Write the equations of all horizontal asymptotes
 - Find the x value of any holes
 - Sketch the graph (no calculator) showing all characteristics listed (Not all functions will have all characteristics listed above)

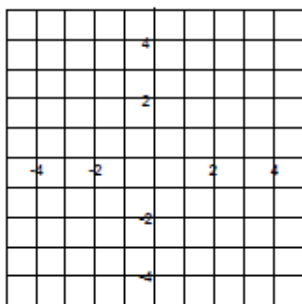
1. $f(x) = \frac{x+2}{x-1}$



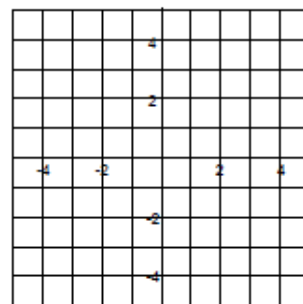
2. $f(x) = \frac{2x-3}{x+1}$



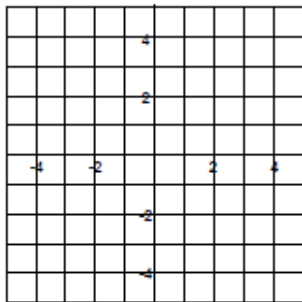
3. $f(x) = \frac{1}{x^2}$



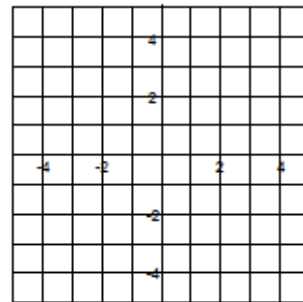
4. $f(x) = \frac{x-1}{x^2}$



5. $f(x) = \frac{x^2 + 3x - 4}{x^2 - x - 6}$



6. $f(x) = \frac{x}{x^2 - 4}$



V. Properties of Natural Logarithms

Recall that $y = \ln(x)$ and $y = e^x$ (exponential function) are inverse to each other

Properties of the Natural Log:

$$\ln(AB) = \ln(A) + \ln(B)$$

$$\text{Ex: } \ln(2) + \ln(5) = \ln(10)$$

$$\ln\left(\frac{A}{B}\right) = \ln(A) - \ln(B)$$

$$\text{Ex: } \ln(6) - \ln(2) = \ln\left(\frac{6}{2}\right) = \ln(3)$$

$$\ln(A^p) = p \ln(A)$$

$$\text{Ex: } \ln(x^4) = 4 \ln(x) \quad \text{and} \quad 3 \ln(2) = \ln(2^3) = \ln(8)$$

Know these without a calculator: $\ln(e^x) = x$, $\ln(e) = 1$, $\ln(1) = 0$, $e^0 = 1$

Use the properties of natural logs to solve for x .

Ex:
 $5^x = 7e^x$

1. $10 = 4^x$

2. $2e^{3x} = 4e^{5x}$

3. $10^{x+3} = 5e^{7-x}$

$$\frac{5^x}{7} = \ln e^x$$

$$\ln\left(\frac{5^x}{7}\right) = \ln e^x$$

$$\ln(5^x) - \ln(7) = x$$

$$x \ln(5) - \ln(7) = x$$

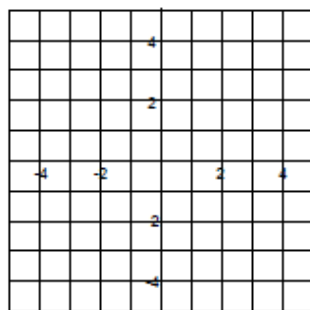
$$x \ln(5) - x = \ln(7)$$

$$x(\ln(5) - 1) = \ln(7)$$

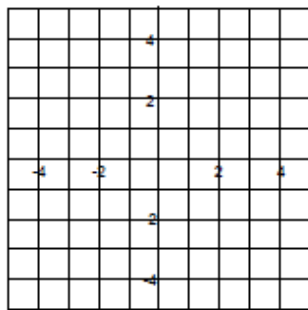
$$x = \frac{\ln(7)}{\ln(5) - 1}$$

Draw a sketch of each function. Make sure to label the key point and an asymptote.

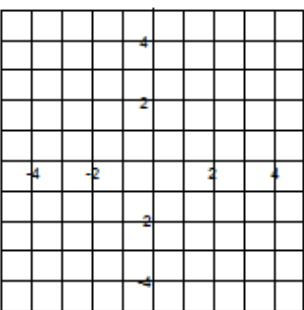
4. $y = 2^x$



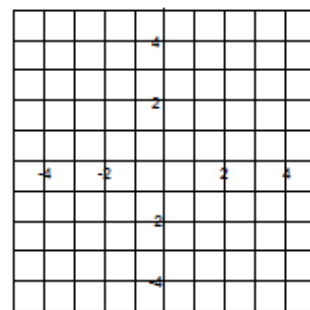
5. $y = e^x$



6. $y = -e^x$



7. $y = \ln x$



VI Calculator Skills

No matter what type of calculator you own, you should be able to do the following problems. Use the index of the owner's manual of your calculator to look up instructions or ask a classmate how to do each.

Evaluations:

1) Fractions and more. Be careful to use enough parentheses.

a) $\frac{6 - .379}{3 + \sqrt{2}}$

b) $\sqrt{2 + \pi}$

2) Logarithms and Exponents

a) $\ln 2$

b) $\log 2$

c) $\ln(3e)$

d) 5^{-4}

e) e^3

f) $\sqrt[3]{10}$

f) -18^2

g) $(-18)^2$

h) $3^{(2)(.0134)}$

3) Trigonometry Evaluations. Make sure you watch the mode – degree versus radians.

a) $\sin 23^\circ$

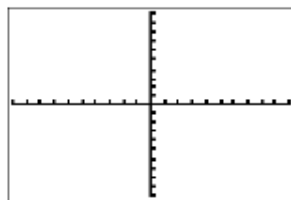
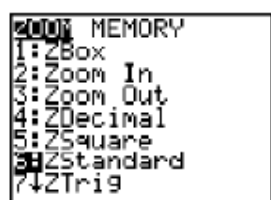
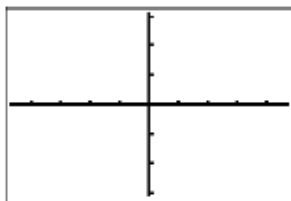
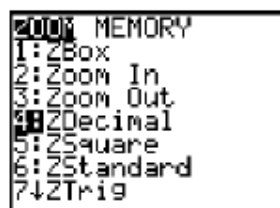
b) $\sin \frac{\pi}{5}$

c) $\sec \frac{\pi}{7}$

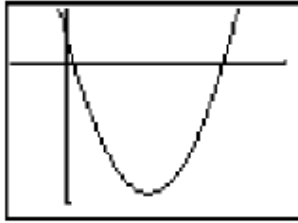
d) $\tan^{-1}(1.5)$ in radians & degrees

Graphing Skill #1: You should be able to graph a function in a viewing window that shows the important features. You should be familiar with the built-in zoom options for setting the window such as zoom-decimal and zoom-standard. You should also be able to set the window conditions to values you choose.

1. Graph $y = x^2 - 3$ using the built in zoom-decimal and zoom-standard options. Draw each.



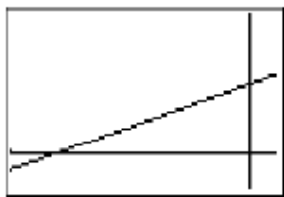
2. Find the appropriate viewing window to see the intercepts and the vertex defined by $y = x^2 - 11x + 10$. Use the window editor to enter the x and y values.



Window: Xmin = _____
 Xmax = _____
 Xscl = _____
 Ymin = _____
 Ymax = _____
 Yscl = _____

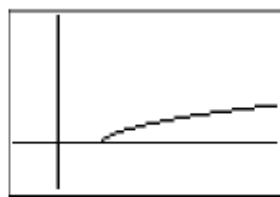
3. Find the appropriate viewing windows for the following functions:

$$y = 10 + \frac{1}{4}x$$



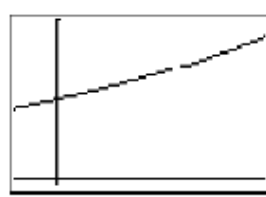
Xmin = _____
 Xmax = _____
 Xscl = _____
 Ymin = _____
 Ymax = _____
 Yscl = _____

$$y = \sqrt{x-5}$$



Xmin = _____
 Xmax = _____
 Xscl = _____
 Ymin = _____
 Ymax = _____
 Yscl = _____

$$y = 100(1.06)^x$$



Xmin = _____
 Xmax = _____
 Xscl = _____
 Ymin = _____
 Ymax = _____
 Yscl = _____

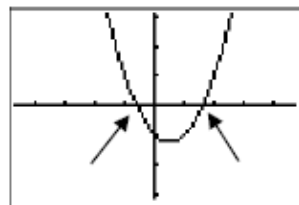
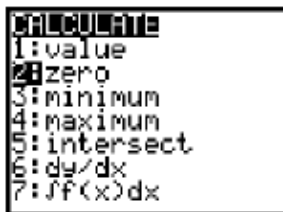
$$y = \frac{1}{x+10}$$



Xmin = _____
 Xmax = _____
 Xscl = _____
 Ymin = _____
 Ymax = _____
 Yscl = _____

Graphing Skill #2: You should be able to graph a function in a viewing window that shows the x-intercepts (also called roots and zeros). You should be able to accurately estimate the x-intercepts to **3 decimal places**. Use the built-in root or zero command.

1. Find the x-intercepts of $y = x^2 - x - 1$. Window $[-4.7, 4.7] \times [-3.1, 3.1]$



(Write intercepts as points)

x-intercepts: _____

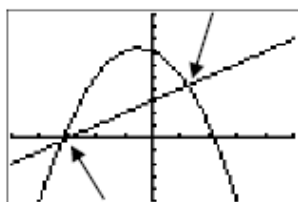
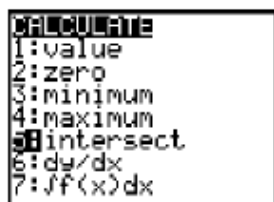
2. Find the x-intercepts of $y = x^3 - 2x - 1$.

x-intercepts: _____

Graphing Skill #3: You should be able to graph two functions in a viewing window that shows the intersection points. Sometimes it is impossible to see all points of intersection in the same viewing window. You should be able to accurately estimate the coordinates of the intersection points to **3 decimal places**. Use the built-in intersection command.

1. Find the coordinates of the intersection points for the functions:

$$f(x) = x + 3 \quad g(x) = -x^2 - x + 7.$$



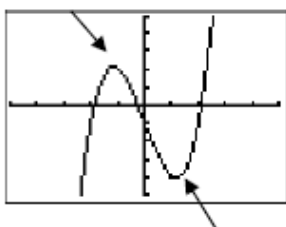
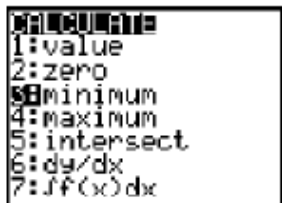
Intersection points: _____

2. Find the coordinates of the intersection points of: $f(x) = 4x^2$ $g(x) = 2^x$.

Intersection points: _____

Graphing Skill #4: You should be able to graph a function and estimate the local maximum or minimum values to **3 decimal places**. Use the built-in max/min command.

1. Find the maximum and minimum values of the function $y = x^3 - 4x - 1$.



(Value means the y-value)

Minimum value: _____

Maximum value: _____

2. Find the maximum and minimum values of the function $y = x^3 - 4x^2 + 4x$.

REVIEW: Do the following problems on your calculator.

1. $\frac{\sqrt{-4 + 8.556}}{2 + \sqrt[3]{9}}$

2. $\ln 4e^{\sqrt{2}} - 1.32$

3. $\sin 48^\circ$

4) $\cos^{-1}(-.75)$ in radians and degrees

5) $\tan \frac{5\pi}{6}$

6) Find the x-intercepts, relative maximum, and relative minimum of $y = x^3 + 2x^2 - 1$

7) Find the coordinates of the intersection points for the functions $f(x) = 2x^2 + x - 9$ and $g(x) = -\frac{3}{4}x + 3$